BURNING RATE FOR A SOLID PROPELLANT RAMJET DEVELOPING CONSTANT THRUST

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BURNING RATE FOR A SOLID PROPELLANT
RAMJET DEVELOPING CONSTANT THRUST

Thesis by

Paul Edward Arbo

Lieutenant, U. S. Navy

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SUMMARY

An analysis of a solid propellant ramjet which carries the propellant in the combustion chamber was made, and a procedure outlined for calculating the required burning rate of such a ramjet which develops constant thrust. The two factors which influenced this development were:

1) The solid propellant ramjet's combustion chamber varies in size with time due to burning of the fuel.

2) The solid propellant ramjet develops constant thrust.

These factors were coordinated with the analysis of the internal flow system of the ramjet, and an expression for the required burning rate was derived.

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SYMBOLS

A = area, square feet

a = velocity of sound, feet per second

C_{D_f} = drag coefficient of combustion chamber

$C_{D_{fi}}$ = drag coefficient for incompressible flow

C_f = skin friction coefficient

C_p = specific heat at constant pressure (for air 0.24 BTU/lb/°F)

d = throat diameter, inches

D_c = internal diameter between individual grains, or charges, making up combustion surface, inches

D_f = drag force due to friction in duct

D_{fi} = incompressible drag force due to friction in duct

E = system parameter, $E = F/\gamma p_o A_t$

f = fuel - air ratio

F = thrust, pounds

g = acceleration of gravity (32.2 ft/sec^2)

h = heating value of fuel, BTU per pound

ΔH = heat added, BTU per pound

I = specific impulse, seconds

L = length of combustion chamber, inches

m = mass flow rate of air, pounds per second

m_F = fuel mass flow rate, pounds per second

M = mach number (v/a)

N = number of individual grains making up combustion surface

p = static pressure, pounds per square inch absolute

q = dynamic pressure, pounds per square inch absolute

R = gas constant, feet per °F (for air 53.3)

S = specific fuel consumption, weight of fuel consumed per hour per pound of thrust

S_o = Ideal specific fuel consumption, weight of fuel consumed per hour per pound of thrust

t_b = time of burning, seconds

T = temperature, °F absolute

T_t = total temperature, °F absolute

v = velocity within duct, feet per second

w_F = weight of fuel consumed, pounds

γ = ratio of specific heats (for air 1.40)

ρ = density, pounds per cubic foot

Subscripts

o to 4 = stations in internal flow system shown in Figure 1

F = fuel

t = throat

I. INTRODUCTION

The use of solid fuel in a ramjet introduces a new problem. As the fuel burns, the effective combustion chamber area increases. This would normally result in a change in thrust; however, if the condition of constant thrust is imposed, then this change in combustion chamber area would vary the amount of heat required. The change in heat required may be expressed in terms of the burning rate required for a solid propellant ramjet developing constant thrust.

References 1 and 2, which are representative of the numerous analyses in this field, were concerned with the internal flow through a ramjet having a constant combustion chamber area, and no restriction on the thrust. These analyses were developed for subsonic conditions, and a limited heat addition.

In the present investigation, a theoretical analysis was made for supersonic conditions of a solid fuel ramjet, which carries the propellant in the combustion chamber, developing constant thrust. This latter requirement fixes the exit conditions and permits calculation of the internal flow from the rear of the ramjet to the combustion chamber inlet. In this manner, the pressure loss across the combustion chamber and nozzle was computed. The pressure rise across the diffuser may be determined from the known inlet conditions. By equating the pressure rise to the pressure loss, unique determination of the heat required for constant thrust can be made. The amount of heat developed is a function of the fuel and

its distribution, and from this, the required burning rate may be determined.

Results of this analysis are presented in Part II of the investigation in the form of working equations, some of which have been solved and presented on charts. These equations and charts have been consolidated, and the required burning rate or specific fuel consumption is presented in Figure 10. Part III illustrates the method of solving a numerical problem.

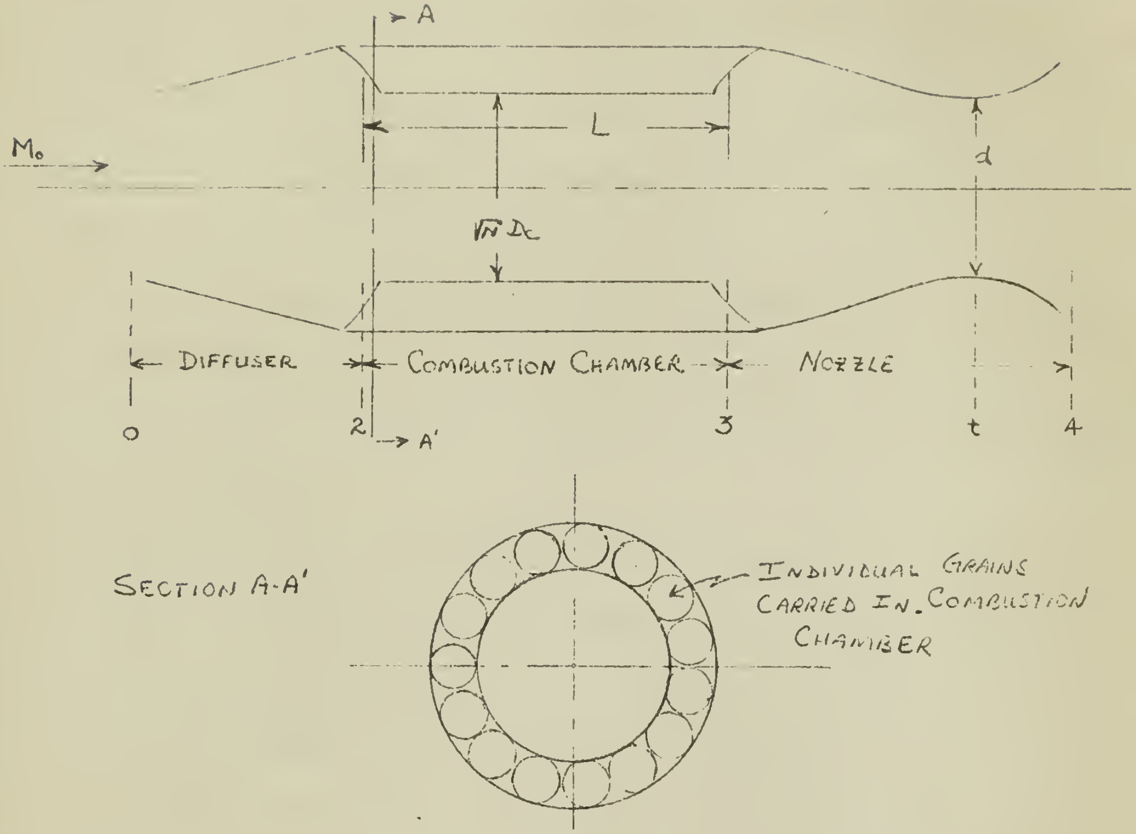


Figure 1 Schematic of a solid propellant ramjet which carries its fuel in the combustion chamber.

II. ANALYSIS OF INTERNAL FLOW SYSTEMS

Flow characteristics of the internal flow system must be calculated so that the state of air entering the combustion chamber, the flow change across the combustion chamber, the net drag of the internal flow, and the required geometry of the ramjet can be determined. In determining the internal flow from the rear end of the ramjet, the following relationships must be established: (A) nozzle throat condition to ambient condition; (B) flow changes across the nozzle; and (C) flow changes across the combustion chamber. These expressions must then be related to (D) flow changes across the diffuser.

One dimensional flow, uniform velocity distribution across the duct, constant gas properties, and ideal expansion were assumed.

Initial quantities assumed to be known are as follows:

(1) The desired thrust at designed altitude.

(2) The heating value and density of the fuel.

(3) Designed Mach number (supersonic) and atmospheric conditions p_0, ρ_0, T_0.

(4) Skin friction coefficient.

(5) Area of the throat of the nozzle.

(6) Ratio of length of chamber to throat diameter.

(7) The number of grains (N) of fuel.

A complete investigation should include consideration of: 1) isentropic diffusion, and 2) normal shock wave; and whether the nozzle is sonic or not at the throat. This analysis outlines in detail the case of isentropic diffusion with a sonic nozzle.

A. Flow Relation Between Nozzle Exit and Ambient Conditions

An exact relation between nozzle exit condition and that of the ambient surroundings may be obtained from the expression for thrust. The thrust of a ramjet is given by the increase in momentum of the internal flow, or

$$F = m\ (V_4 - V_0).$$

The mass rate of flow at the nozzle throat is

$$m = \rho_t\ V_t\ A_t = \rho_t\ a_t\ A_t \quad \text{since } M_t = 1$$

The thrust may then be expressed as

$$F = \rho_t\ a_t\ A_t\ (V_4 - V_0)$$

$$= \frac{P_t A_t}{R\,T_t} \sqrt{\gamma R T_t} \left(M_4 \sqrt{\gamma R T_4} - M_0 \sqrt{\gamma R T_0} \right)$$

or

$$\frac{F}{\gamma P_0 A_t} = \frac{P_t}{P_0} \left(M_4 \sqrt{\frac{T_4}{T_t}} - M_0 \sqrt{\frac{T_0}{T_t}} \right)$$

$$= \frac{P_t}{P_0} \sqrt{\frac{T_4}{T_t}} \left(M_4 - M_0 \sqrt{\frac{T_0}{T_4}} \right)$$

All of the factors on the left hand side of the equation are known initially, therefore a new parameter, E, is introduced

$$E = \frac{F}{\gamma P_0 A_\tau}$$

Then by the perfect gas law

$$\frac{P_\tau}{P_4} = \frac{P_\tau}{P_0} = \left(\frac{T_4}{T_\tau}\right)^{-\frac{\gamma}{\gamma-1}}$$

and

$$\frac{P_\tau}{P_0}\sqrt{\frac{T_4}{T_\tau}} = \left(\frac{T_4}{T_\tau}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

Using the isentropic relation

$$\frac{T_4}{T_\tau} = \frac{\frac{\gamma+1}{2}}{1 + \frac{\gamma-1}{2} M_4^2}$$

then

$$E = \left[\frac{\frac{\gamma+1}{2}}{1 + \frac{\gamma-1}{2} M_4^2}\right]^{-\frac{\gamma+1}{2(\gamma-1)}} \left[M_4 - M_0\sqrt{\frac{T_0}{T_4}}\right] \tag{1}$$

Solving for $\frac{T_0}{T_4}$,

$$\frac{T_0}{T_4} = \left[\frac{M_4 - E\left[\frac{\frac{\gamma+1}{2}}{1 + \frac{\gamma-1}{2} M_4^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}{M_0}\right]^2 \tag{1A}$$

T_4 is not a known parameter, but it can be related to the temperature rise. The heat liberated is equal to the total temperature rise, or

$$\frac{T_{3t} - T_{2t}}{T_{2t}} = \frac{\Delta H}{C_p T_{2t}} \tag{2}$$

The compression from free stream to station 2 at the combustion chamber inlet is an isentropic one, and the state 2 is assumed to be a stagnation state, where

$$T_{0t} = T_{2t}$$

Similarly the expansion from state 3 to state 4 is an isentropic one and

$$T_{3t} = T_{4t}$$

Equation (2) becomes

$$\frac{T_{4t}}{T_{0t}} = 1 + \frac{\Delta H}{C_p T_{0t}} \tag{3}$$

Using the isentropic relation

$$T_{4t} = T_4 \left(1 + \frac{\gamma-1}{2} M_4^2\right)$$

equation (3) becomes

$$\frac{T_0}{T_4} = \left(\frac{1}{1 + \frac{\Delta H}{C_p T_{0t}}}\right)\left(\frac{1 + \frac{\gamma-1}{2} M_4^2}{1 + \frac{\gamma-1}{2} M_0^2}\right) \tag{4}$$

Equating (1A) and (4), and solving for $\frac{\Delta H}{C_p T_{0t}}$, results in the following expression,

$$\frac{\Delta H}{C_p T_{0t}} = \left[\frac{M_0}{M_4 - E\left[\dfrac{\frac{\gamma+1}{2}}{1 + \frac{\gamma-1}{2} M_4^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}\right]^2 \left(\frac{1 + \frac{\gamma-1}{2} M_4^2}{1 + \frac{\gamma-1}{2} M_0^2}\right) - 1 \tag{5}$$

This equation can be solved in the form of a graph and would be similar to that shown in Figure 2.

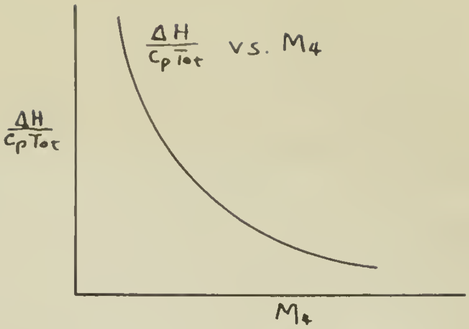


Figure 2

The ideal value of heat required $\left(\frac{\Delta H}{C_p T_{ot}}\right)_{ideal}$ for a ramjet with no losses can be obtained by entering Figure 2 with $M_4 = M_0$. As losses develop, the exit mach number M_4 decreases, and a larger value of $\frac{\Delta H}{C_p T_{ot}}$ is required to maintain constant thrust.

B. Flow Changes Across The Nozzle

In development of the analysis from the rear of the ramjet, the flow across the nozzle must be expressed in terms of the combustion chamber outlet. The pressure ratio across the nozzle is

$$\frac{P_3}{P_4} = \frac{P_3}{P_o} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma-1}} \tag{6}$$

or, in terms of mach numbers,

$$\frac{P_3}{P_o} = \left(\frac{1 + \frac{\gamma-1}{2} M_4^2}{1 + \frac{\gamma-1}{2} M_3^2}\right)^{\frac{\gamma}{\gamma-1}}$$

hence

$$\frac{T_3}{T_4} = \frac{1 + \frac{\gamma-1}{2} M_4^2}{1 + \frac{\gamma-1}{2} M_3^2} \tag{7}$$

By the continuity equation

$$\dot{m}_3 = \dot{m}_t$$

$$\rho_3 a_3 M_3 \frac{N\pi D_c^2}{4} = \rho_t a_t M_t \frac{\pi d^2}{4}$$

or

$$M_3 = \frac{\rho_t a_t}{\rho_3 a_3}\left(\frac{d}{\sqrt{N} D_c}\right)^2$$

Also,

$$M_3 = \left(\frac{T_4}{T_3}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left(\frac{T_t}{T_4}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left(\frac{d}{\sqrt{N} D_c}\right)^2$$

Solving for $\frac{T_3}{T_4}$.

$$\frac{T_3}{T_4} = \left(\frac{1+\frac{\gamma-1}{2}M_4^2}{\frac{\gamma+1}{2}}\right)\left[\frac{1}{M_3}\left(\frac{d}{\sqrt{N} D_c}\right)^2\right]^{\frac{2(\gamma-1)}{\gamma+1}} \tag{8}$$

Substitution of this relation into equation (6) yields

$$\frac{P_3}{P_0} = \left(\frac{1+\frac{\gamma-1}{2}M_4^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma}{\gamma-1}}\left[\frac{1}{M_3^2}\left(\frac{d}{\sqrt{N} D_c}\right)^4\right]^{\frac{\gamma}{\gamma+1}} \tag{9}$$

A relation between $\frac{d}{\sqrt{N} D_c}$ and M_3 may be obtained by equating (7) and (8)

$$\frac{d}{\sqrt{N} D_c} = \sqrt{M_3\left(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_3^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \tag{10}$$

In equation (9), the term $\frac{d}{\sqrt{N}\,D_c}$ is a known quantity and has physical significance, whereas M_3 is an unknown value and is not generally used as a parameter. This analysis will therefore be developed in terms of $\frac{d}{\sqrt{N}\,D_c}$; however, because of ease of conversion, as seen in equation (10), solutions to all equations were derived in terms of M_3. The final result is expressed in the known parameter, $\frac{d}{\sqrt{N}\,D_c}$.

Equation (9) may be written as

$$\frac{P_3}{P_0} = \left(\frac{1 + \frac{\gamma-1}{2} M_4^2}{1 + \frac{\gamma-1}{2} M_3^2} \right)^{\frac{\gamma}{\gamma-1}} \tag{11}$$

C. Flow Changes Across The Combustion Chamber

The flow through the combustion chamber is considered next. The channel or duct has a constant cross section in which friction losses occur. As the fuel burns uniformly, the cross section of the duct increases, and a change in heat added is required to maintain constant thrust. In ~~keeping~~ with the procedure outlined, all relations will be stated in terms of combustion chamber outlet conditions.

The pressure drop between stations 2 and 3 (at combustion chamber inlet and outlet) is obtained by applying the momentum relation between these stations:

$$P_2 + \rho_2 V_2^2 = P_3 + \rho_3 V_3^2 + \frac{D_f}{A_3} \tag{12}$$

where D_f is the drag force due to friction within the combustion chamber, and $A_2 = A_3$.

From equation (12) it is evident that the pressure drop is composed of a component associated with the frictional drag force, and a component due to the momentum increase in the duct. As pointed out in Reference (1), the former component is frequently neglected but becomes very important when the velocity in the duct is large. The drag term is presented in the form of a drag coefficient, C_{Df} , where

$$C_{Df} = \frac{D_f}{q_2 A_2} \tag{13}$$

If the incompressible flow is taken as a basis for the unheated low speed condition, the drag coefficient becomes

$$C_{Dfi} = \left(\frac{D_f}{q_2 A_2}\right)_i \tag{14}$$

and is determined by the geometry of the ramjet and the Reynolds number. There is an increase in the dynamic pressure when heat is applied, resulting in an increase in the incompressible drag coefficient. If the dynamic pressure varies linearly with the length of the combustion chamber, an average value may be assumed, and equation (14) becomes

$$C_{Dfi} = \frac{D_{fi}}{\frac{1}{2}\left[\frac{\rho_2 V_2^2 + \rho_3 V_3^2}{2}\right] A_2}$$

If the incompressible drag is now expressed in terms of the skin

friction coefficient (a known value), C_{Dfi} becomes

$$C_{Dfi} = \frac{C_f \frac{1}{2}\left[\frac{\rho_2 V_2^2 + \rho_3 V_3^2}{2}\right] L N \pi D_c}{\frac{1}{2}\left[\frac{\rho_2 V_2^2 + \rho_3 V_3^2}{2}\right] \frac{N \pi D_c^2}{4}}$$

or

$$C_{Dfi} = 4 \frac{L}{D_c} C_f$$

hence

$$D_{fi} = 4 \frac{L}{D_c} C_f \frac{1}{2}\left[\frac{\rho_2 V_2^2 + \rho_3 V_3^2}{2}\right] A_3 \qquad (15)$$

Since the heating effect has been accounted for in the basic drag relation, the two drag equations (13) and (15) may be equated

$$C_{Df} \frac{\rho_3 V_3^2}{2} \rho_3/\rho_2 \, A_3 = 4 \frac{L}{D_c} C_f \frac{\rho_3 V_3^2}{4} \left(\rho_3/\rho_2 + 1\right) A_3$$

or

$$C_{Df} = 2 \frac{L}{D_c} C_f \left(1 + \rho_2/\rho_3\right) \qquad (16)$$

With this definition of the drag coefficient, equation (12) becomes

$$p_2 + \rho_2 V_2^2 = p_3 + \rho_3 V_3^2 + 2 \frac{L}{D_c} C_f \frac{\rho_2 V_2^2}{2} \left(1 + \rho_2/\rho_3\right)$$

or

$$\frac{p_2}{p_3} = 1 + \gamma M_3^2 \left[\frac{L}{D_c} C_f \left(1 + \rho_3/\rho_2\right) + \left(1 - \rho_3/\rho_2\right)\right]$$

or, in terms of the diameter ratio, since D_c varies,

$$\frac{p_2}{p_3} = 1 + \gamma M_3^2 \left[\frac{\sqrt{N}\,L}{d} \; \frac{d}{\sqrt{N}\,D_c} C_f \left(1 + \rho_3/\rho_2\right) + \left(1 - \rho_3/\rho_2\right) \right] \tag{17}$$

In order to evaluate the pressure ratio, the density ratio must first be obtained. This ratio may be determined by simultaneous solution of the energy equation,

$$\frac{V_2^2}{2} + C_p T_2 + \Delta H = \frac{V_3^2}{2} + C_p T_3$$

the continuity equation,

$$\rho_2 V_2 = \rho_3 V_3$$

the perfect gas relation,

$$p = \rho R T$$

and the momentum relation, equation (12). The final result is

$$M_3^2 \left[\frac{\gamma+1}{2} - \gamma \frac{L}{D_c} C_f \right] \left(\rho_3/\rho_2\right)^2 - \left[1 + \gamma M_3^2 \left(\frac{L}{D_c} C_f + 1 \right) \right] \left(\rho_3/\rho_2\right) + \frac{1 + \frac{\gamma-1}{2} M_3^2}{1 + \frac{\Delta H}{C_p T_{0\tau}}} = 0$$

or in terms of $\frac{d}{\sqrt{N}\,D_c}$,

$$M_3^2 \left[\frac{\gamma+1}{2} - \gamma \frac{\sqrt{N}\,L}{d} \frac{d}{\sqrt{N}\,D_c} C_f \right] \left(\rho_3/\rho_2\right)^2 - \left[1 + \gamma M_3^2 \left(\frac{\sqrt{N}\,L}{d} \frac{d}{\sqrt{N}\,D_c} C_f + 1 \right) \right] \left(\rho_3/\rho_2\right) + \frac{1 + \frac{\gamma-1}{2} M_3^2}{1 + \frac{\Delta H}{C_p T_{0\tau}}} = 0 \tag{18}$$

Solution of equation (17) provides $\frac{p_2}{p_0}$, for,

$$\frac{p_2}{p_0} = \frac{p_3}{p_0} \cdot \frac{p_2}{p_3} \tag{19}$$

Thus the pressure drop across the combustion chamber and nozzle has been calculated, and must equal the pressure rise across the diffuser.

Solution of equation (17) also permits determination of M_2, for by the continuity equation,

$$\rho_2 a_2 M_2 = \rho_3 a_3 M_3$$

or

$$\left(\frac{M_3}{M_2}\right)^2 = \left(\rho_2/\rho_3\right)^2 \frac{T_2}{T_3}$$

Also,

$$\frac{T_3}{T_2} = \frac{T_{3t}}{T_{2t}} \left(\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_3^2}\right) = \left(1 + \frac{\Delta H}{c_p T_{0t}}\right)\left(\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_3^2}\right)$$

Combining the last two equations, and simplifying,

$$M_2^2 = \frac{1}{\dfrac{1 + \frac{\gamma-1}{2} M_3^2}{M_3^2\left(1 + \frac{\Delta H}{c_p T_{0t}}\right)}\left(\rho_2/\rho_3\right)^2 - \dfrac{\gamma-1}{2}} \tag{20}$$

The pressure rise across the diffuser as computed from the rear of the ramjet was plotted vs the combustion chamber inlet mach number as calculated in equation (20), (see Figure 3).

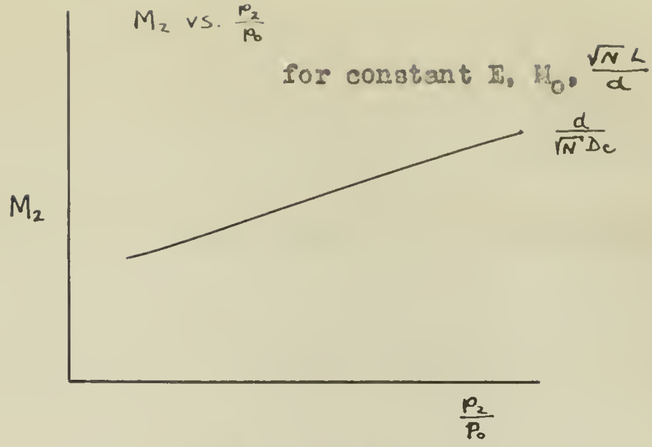


Figure 3

A plot of combustion chamber inlet mach number and heat required for constant thrust as developed in equations (18) and (20) is shown in Figure 4.

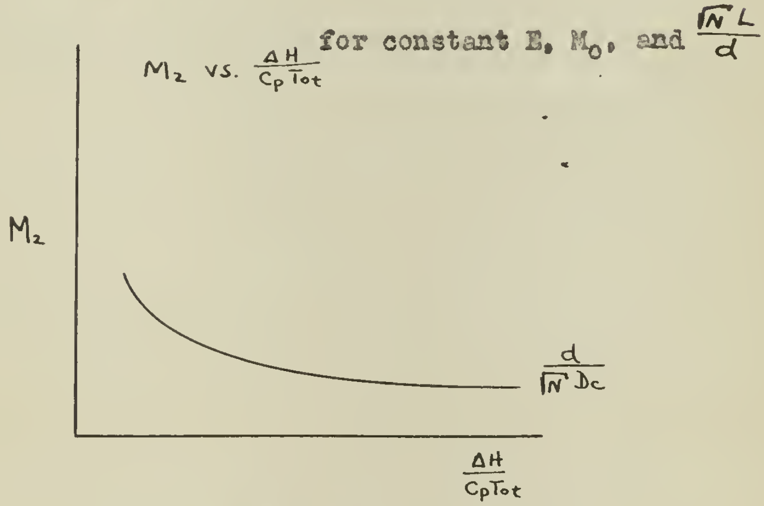


Figure 4

D. Flow Changes Across The Diffuser

In an analysis of a ramjet having a designed supersonic mach number, there are two types of diffusion to be considered - namely, isentropic and normal shock wave diffusion. In this investigation, diffusion was considered to be isentropic, and the diffuser 100% efficient. The pressure rise across the diffuser is

$$\frac{P_2}{P_0} = \left(\frac{T_2}{T_0}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}} \qquad (21)$$

Now, the pressure rise across the diffuser as computed from the front end of the ramjet was plotted in Figure 5.

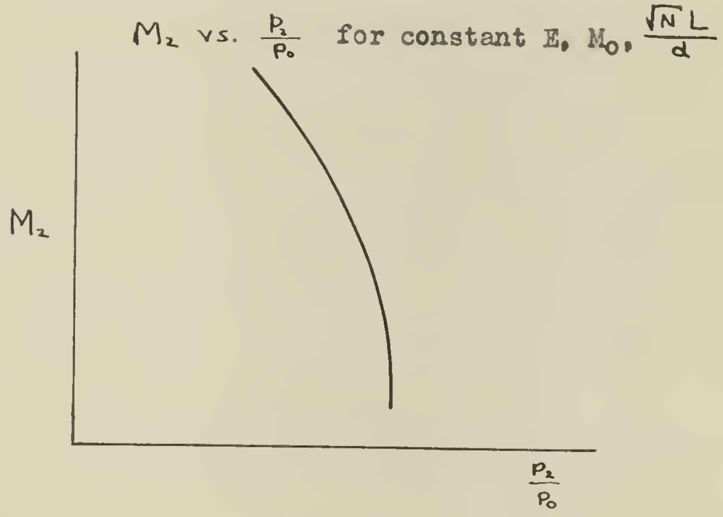


Figure 5

Unique determination of M_2 and $\frac{P_2}{P_0}$ for constant thrust was obtained by superimposing Figure 5 on Figure 3 and

noting the points of intersection as shown in Figure 6.

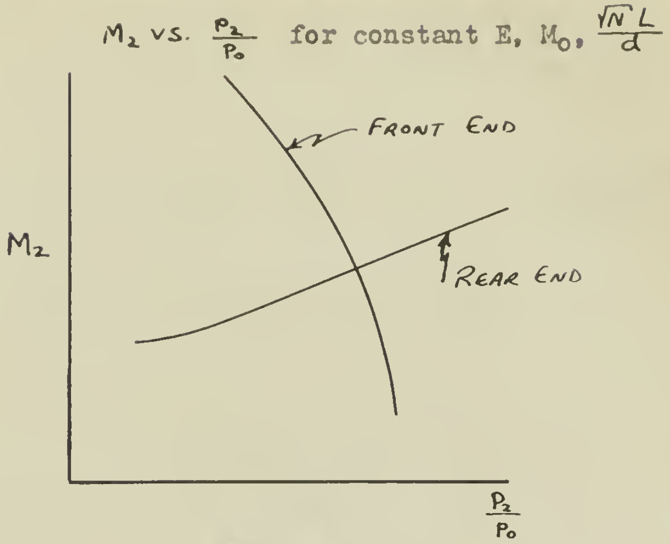


Figure 6

E. Determination of Burning Rate

The condition of constant thrust has been satisfied by the simultaneous solution of equations (19), (20), and (21). With the value of M_2 obtained at the intersection of the two curves as a parameter, the value of heat required $\left(\frac{\Delta H}{C_p T_{0t}}\right)$ was determined from Figure 4 for a given value of $\frac{d}{\sqrt{N} D_c}$. After obtaining values of $\frac{\Delta H}{C_p T_{0t}}$ for various $\frac{d}{\sqrt{N} D_c}$, a chart was next constructed of $\frac{d}{\sqrt{N} D_c}$ vs. $\frac{\Delta H}{C_p T_{0t}}$ (see Figure 7).

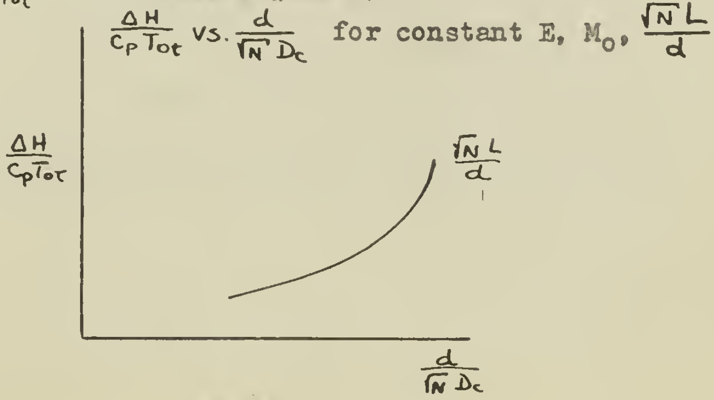


Figure 7

This chart was used later to obtain the heat required for an elemental volume. In considering an elemental volume, the weight of fuel consumed and the time of consumption must be calculated.

To accomplish this, the mass rate of flow is written as

$$m = \rho_2 a_2 M_2 A_2 = \rho_0 a_0 M_2 \frac{N\pi D_c^2}{4} \frac{\rho_2 a_2}{\rho_0 a_0}$$

where

$$\frac{\rho_2 a_2}{\rho_0 a_0} = \left(\frac{1 + \frac{\gamma-1}{2} M_0^2}{1 + \frac{\gamma-1}{2} M_2^2} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

Then,

$$m = \rho_0 a_0 M_2 A_t \left(\frac{\sqrt{N}\, D_c}{d} \right)^2 \left(\frac{1 + \frac{\gamma-1}{2} M_0^2}{1 + \frac{\gamma-1}{2} M_2^2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (22)$$

The fuel-air ratio may be determined from

$$f = \frac{\Delta H}{h}$$

or

$$f = \frac{m_F}{m}$$

The Δ H was obtained from Figure 6, and h is a given value, therefore

$$m_F = m f$$

The final relation between the weight of fuel burned, the time of burning, and the changes in diameter may be obtained by

considering a small elemental volume of fuel. The volume (V_1) at time t is

$$V_1 = L N \pi D_c (\Delta D_c) \tag{23}$$

where ΔD_c is the change in fuel diameter. At time $t + \Delta t$,

$$D_c(t+\Delta t) = D_c(t) + \Delta D_c = D_c(t) + \frac{V_1}{L N \pi D_c(t)} \tag{24}$$

Or, in terms of $\frac{\sqrt{N} D_c}{d}$,

$$\frac{\sqrt{N} D_c(t+\Delta t)}{d} = \frac{\sqrt{N} D_c(t)}{d} + \frac{\sqrt{N} V_1}{L N \pi D_c^2(t) d}$$

$$= \frac{\sqrt{N} D_c(t)}{d} \left(1 + \frac{V_1}{L N \pi D_c^2(t)} \right)$$

Hence, solving for V_1 ,

$$V_1 = L N \pi D_c^2(t) \left[\frac{\frac{\sqrt{N} D_c(t+\Delta t)}{d}}{\frac{\sqrt{N} D_c(t)}{d}} - 1 \right]$$

$$V_1 = 4 L A_t \left(\frac{\sqrt{N} D_c(t)}{d} \right)^2 \left[\frac{\frac{\sqrt{N} D_c(t+\Delta t)}{d}}{\frac{\sqrt{N} D_c(t)}{d}} - 1 \right] \tag{25}$$

The weight of the fuel consumed, W_F , may be obtained from

$$W_F = \rho_F V_1$$

The time of burning of this elemental volume is

$$\Delta t_b = \frac{W_F}{m_F}$$

A plot of the weight of fuel consumed vs. diameter ratio and vs. time of burning was made in Figures 8 and 9.

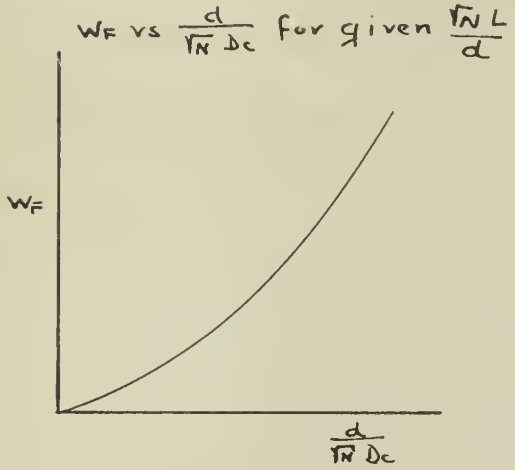

W_F vs $\frac{d}{\sqrt{N}\,D_c}$ for given $\frac{\sqrt{N}\,L}{d}$

Figure 8

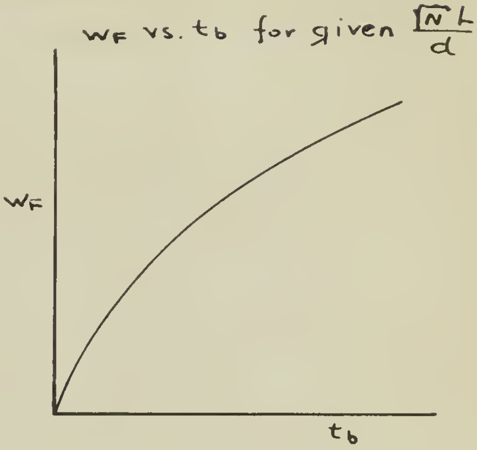

W_F vs. t_b for given $\frac{\sqrt{N}\,L}{d}$

Figure 9

To show the curve in detail, the specific fuel consumption minus the ideal specific fuel consumption was plotted. The specific fuel consumption is

$$S = \frac{3600\,f}{I} = \frac{3600\,m_F}{F} \tag{26}$$

For the ideal case in which there are no losses,

$$m = \frac{\gamma P_o A_t}{a_o}\,\frac{1}{\sqrt{1+\frac{\Delta H}{c_p T_{ot}}}}\left(\frac{1+\frac{\gamma-1}{2}M_v^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{27}$$

The value of $\frac{\Delta H}{C_p T_{0t}}$ is obtained from Figure 2 entering with $M_4 = M_0$ as a parameter. Then

$$S_0 = \frac{3600\,\Delta H\, m\, g}{h\, F}$$

where m is as expressed in equation (27).

A plot was then constructed of $(S - S_0)$ vs. time of burning (see Figure 10).

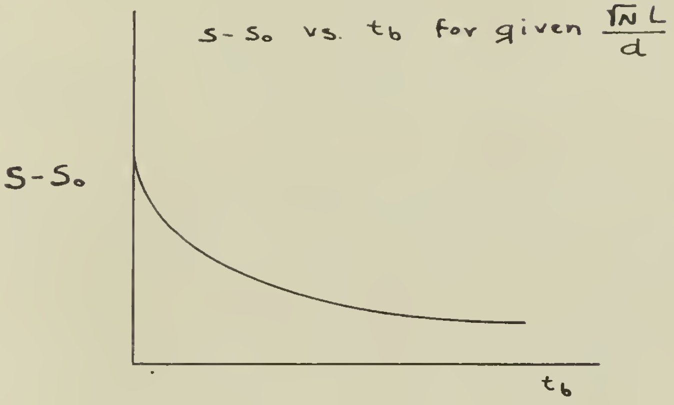


Figure 10

III. A NUMERICAL PROBLEM

In a numerical problem to determine the burning rate of a solid fuel ramjet (which carries its fuel in the combustion chamber) developing constant thrust, the following data is given:

$M_0 = 2$
$Z = 2$
$\gamma \doteq 1.4$
$p_0 = 4.362$ psia.

$h = 14{,}220.65$ BTU/lb.
$\rho_F = 140.5\ \text{lb}/\text{ft}^3$
$A_t = 1\ \text{ft}^2$
$T_0 = 411.7^\circ R$

$N = 25$
$C_f = 0.003$
$\frac{\sqrt{N}\,L}{d} = 50$
$F = 1758$ lb.

Equation (5) has been solved for these values, and is shown in Figure 11. From this chart, the ideal value of heat required for a ramjet with no losses is 1.019. As the losses develop, the exit mach number is decreased and a larger value of $\frac{\Delta H}{c_p T_{oc}}$ is required.

The relation between $\frac{d}{\sqrt{N}\,D_c}$ and M_3 as expressed in equation (10) is shown in Figure 12.

Solutions of equations (19) and (20) for various values of $\frac{\sqrt{N}\,L}{d}$ are shown in Figures 13A, 13B, and 13C. It is to be recalled that these solutions were calculated from the rear of the ramjet. Equation (21), calculated from the front end, is plotted on these same figures, and the intersection of these two curves uniquely determine $\frac{p_2}{p_0}$ and M_2 for a constant thrust, solid propellant ramjet.

The heat required to maintain this constant thrust was plotted vs. the combustion chamber inlet mach number, and is shown in Figures 14A, 14B, and 14C. Using the uniquely determined value of M_2 as the entering parameter, the heat required was obtained as a function of the diameter ratio $\left(\frac{d}{\sqrt{N} D_c}\right)$, and is shown in Figure 15.

Using the process of an elemental volume as described in Section E, and solving equation (25), a relation between the weight of the fuel consumed and the diameter ratio was determined. This relation is shown in Figure 16. The weight of fuel consumed was plotted vs. the burning time in Figure 17. The slope of this curve is the required burning rate. Unfortunately, for the values chosen, the burning rate is relatively constant. Actually, the required burning rate should decrease with an increase in burning time. To show the relation between the weight of fuel consumed and the burning time in greater detail, the specific fuel consumption was calculated from equation (26) and the value of $(S - S_0)$ was plotted vs. time of burning and is shown in Figure 18. This curve shows a change of approximately fifty percent in $(S - S_0)$ over the range of burning time.

REFERENCES

1. J. V. Becker and D. D. Baals, NACA Report No. 773, 1943.

2. I. I. Pinkel and H. Shames, NACA Report No. 880, 1947.

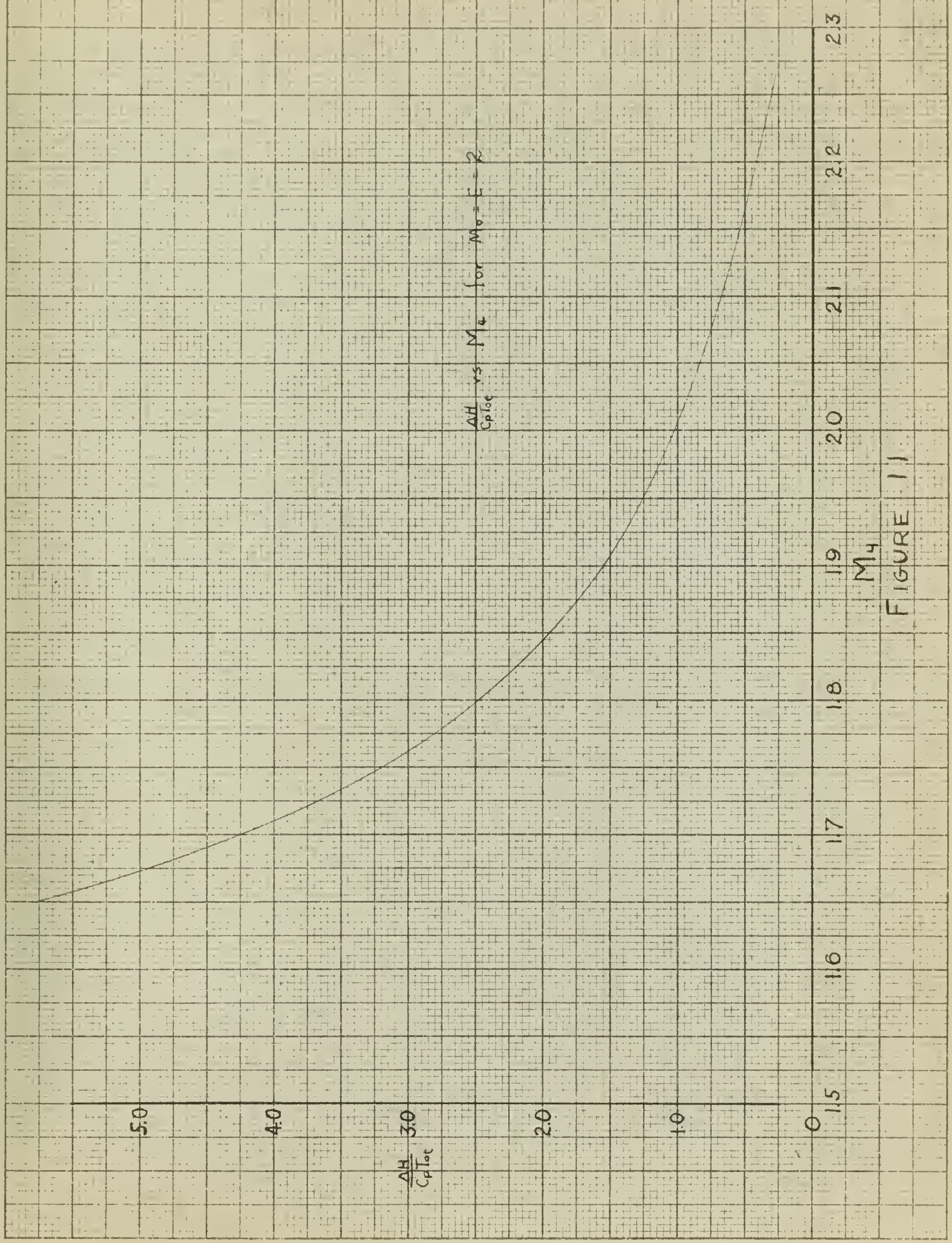

$\frac{\Delta H}{C_p T_{ot}}$ vs M_4 for $M_0 = E - 2$
$\frac{\Delta H}{C_p T_{ot}}$
5.0
4.0
3.0
2.0
1.0
0
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
M_4

FIGURE 11

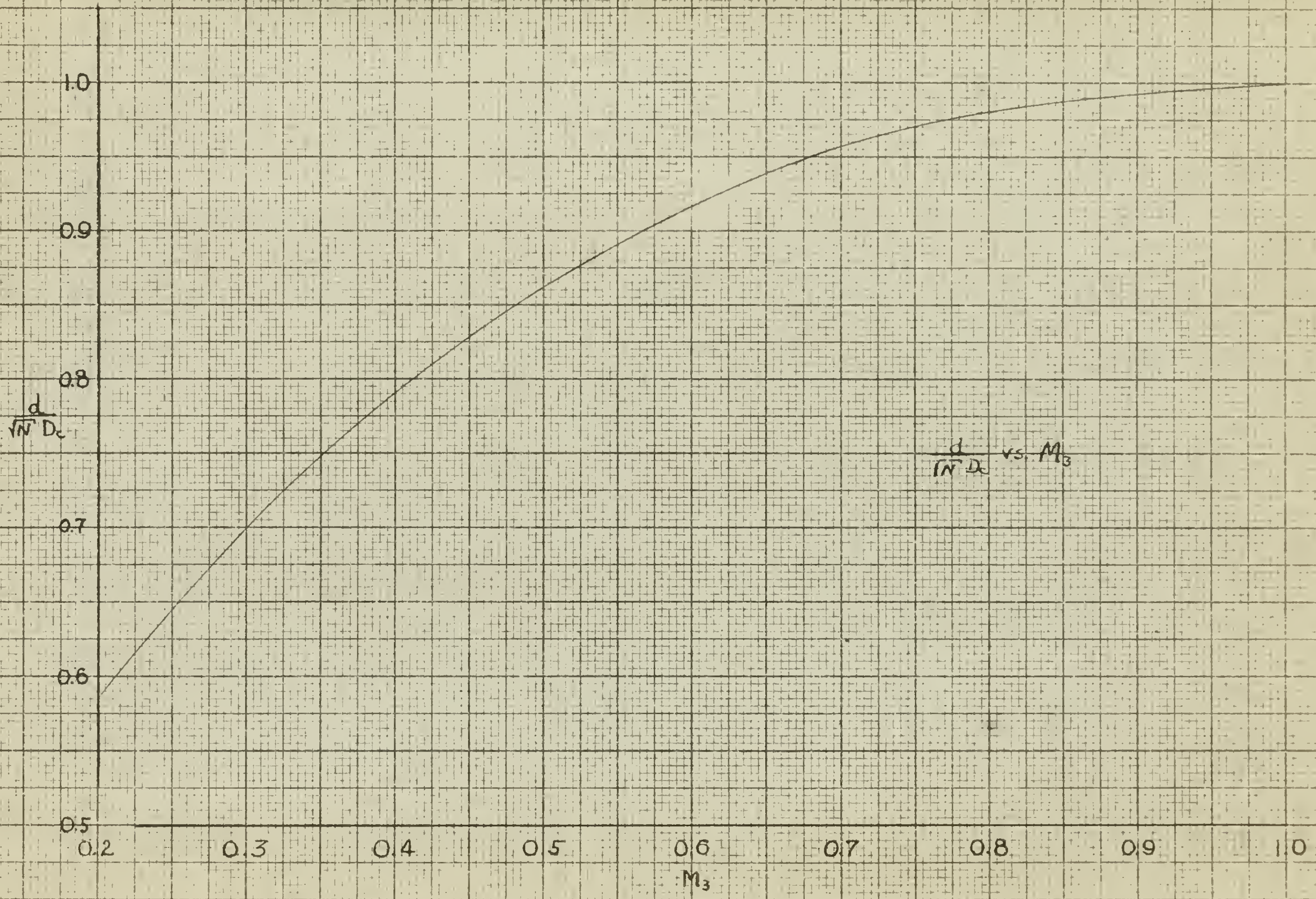

$\frac{d}{\sqrt{N}\,D_c}$
1.0
0.9
0.8
0.7
0.6
0.5
$\frac{d}{\sqrt{N}\,D_c}$ vs. M_3
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
M_3

Figure 12

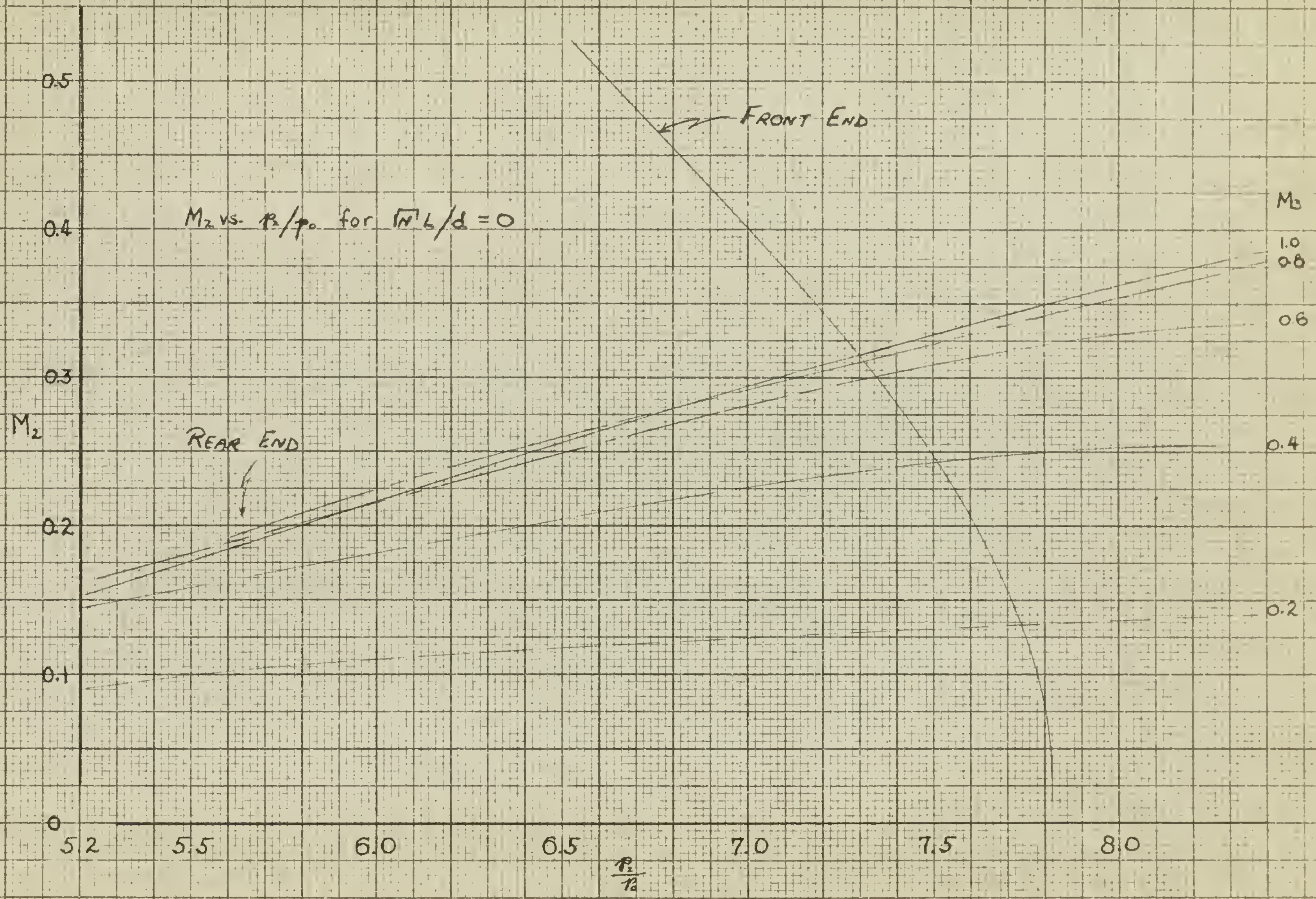
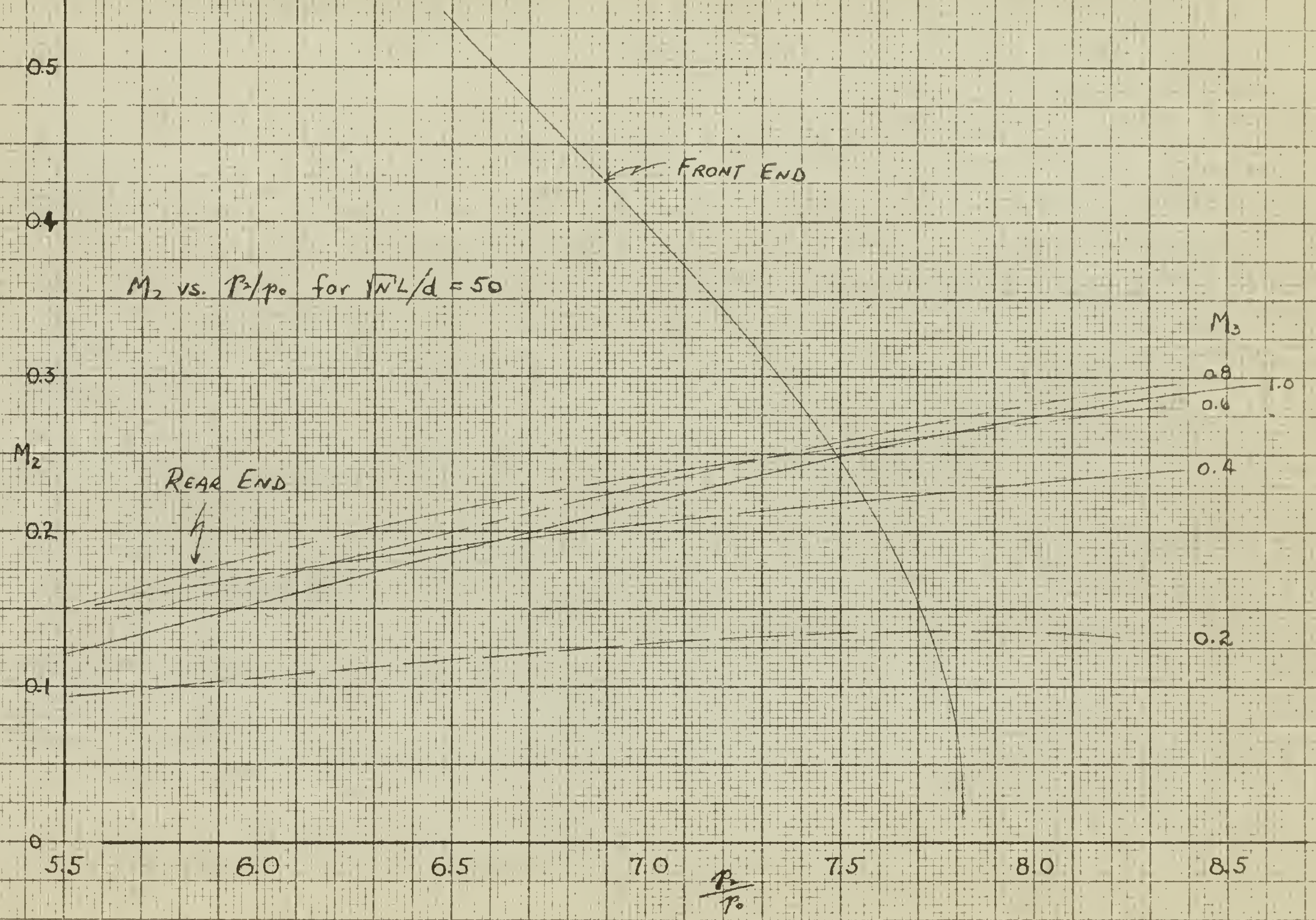

M_2 vs. p_2/p_0 for $\sqrt{N} L/d = 0$
Front End
Rear End
M_3
1.0
0.8
0.6
0.4
0.2
M_2
0.5
0.4
0.3
0.2
0.1
0
5.2
5.5
6.0
6.5
7.0
7.5
8.0
$\frac{p_2}{p_0}$

FIGURE 13A



M_2 vs. p_3/p_0 for $\sqrt{N} L/d = 50$

FIGURE 13.B

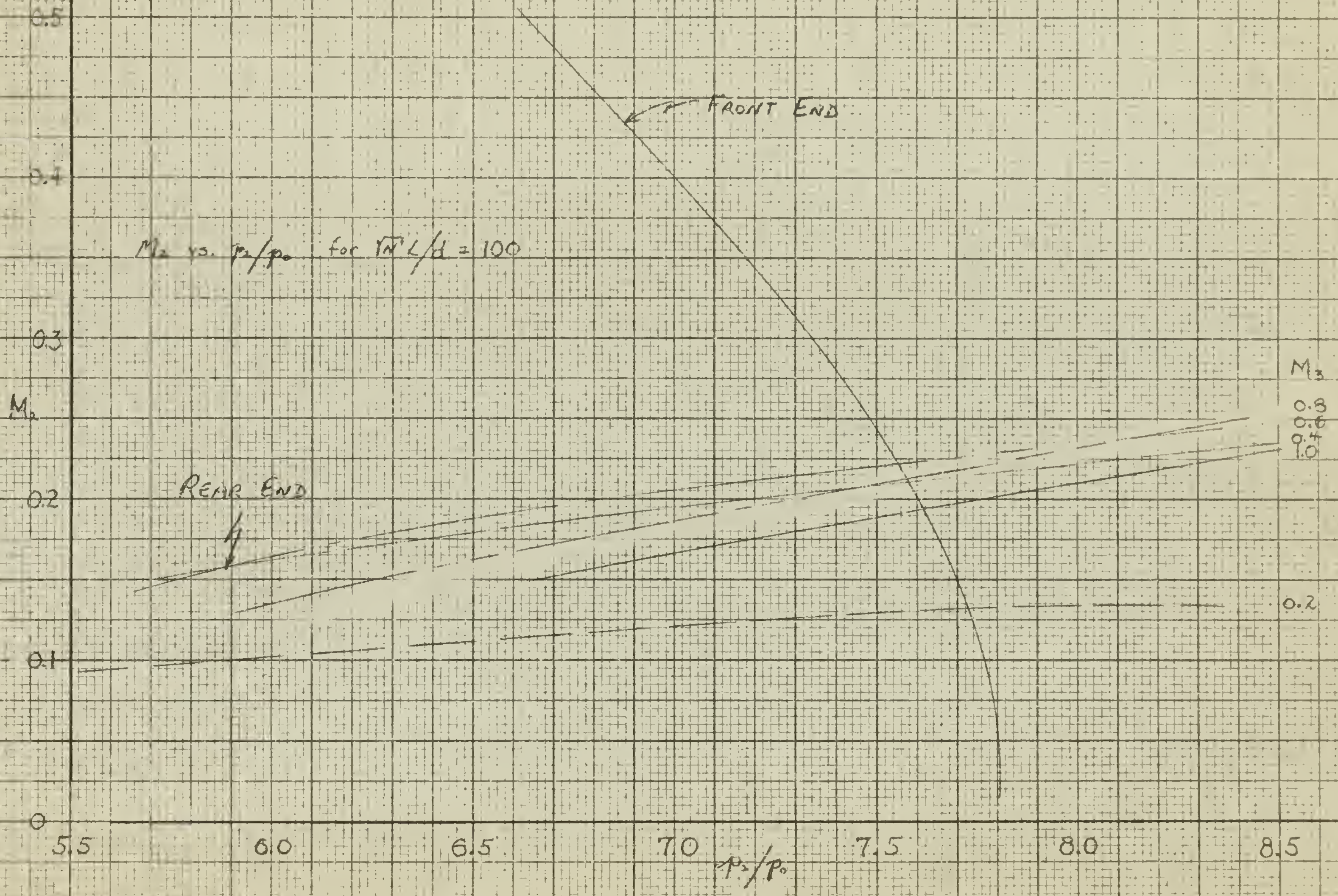
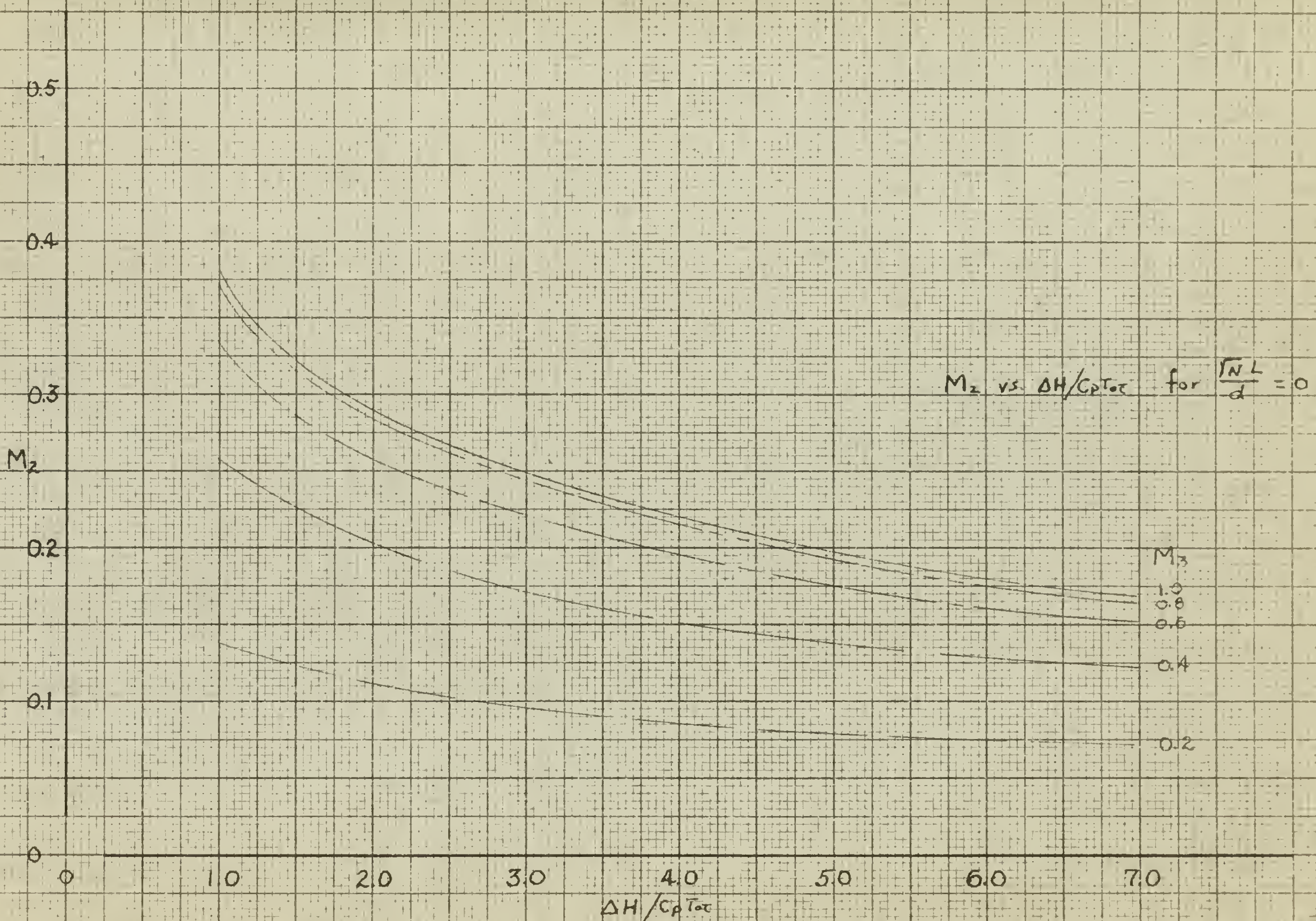

M_2 vs. p_2/p_0 for $\sqrt{N}\,L/d = 100$
FRONT END
REAR END
M_3
0.8
0.6
0.4
1.0
0.2
M_2
0.5
0.4
0.3
0.2
0.1
0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
p_2/p_0

FIGURE 13C



M_2 vs. $\Delta H / C_p T_{0\tau}$ for $\frac{\sqrt{N}\,L}{d} = 0$

FIGURE 14A

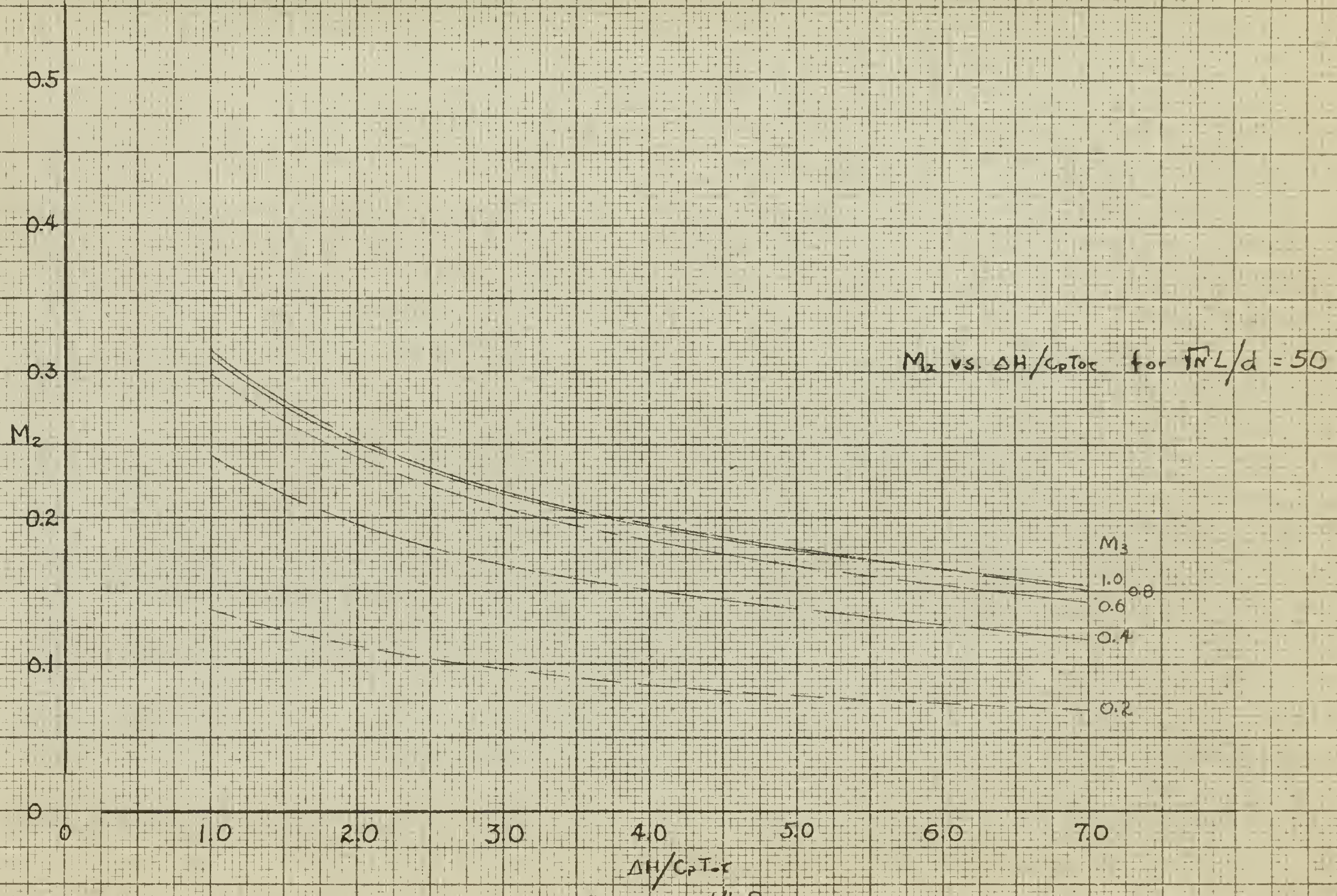

M_2 vs $\Delta H/C_pT_{oc}$ for $\sqrt{f_N}\,L/d = 50$
M_2
0.5
0.4
0.3
0.2
0.1
0
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
$\Delta H/C_pT_{oc}$
M_3
1.0
0.8
0.6
0.4
0.2

Figure 14 B

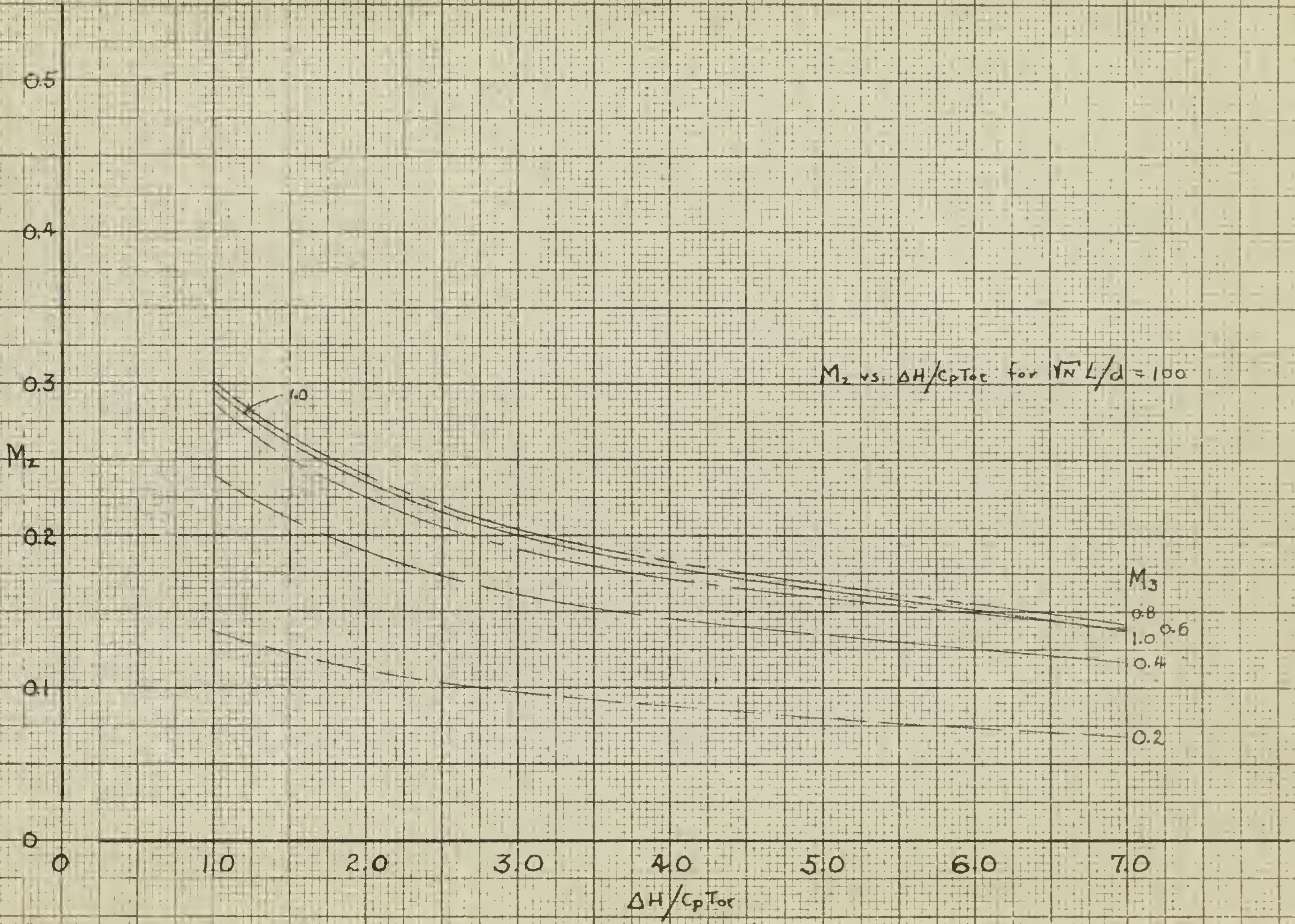

M_2 vs. $\Delta H/c_pT_{oc}$ for $\sqrt{N}\,L/d = 100$
M_2
0.5
0.4
0.3
0.2
0.1
0
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
$\Delta H/c_pT_{oc}$
1.0
M_3
0.8
1.0
0.6
0.4
0.2

FIGURE 14 C

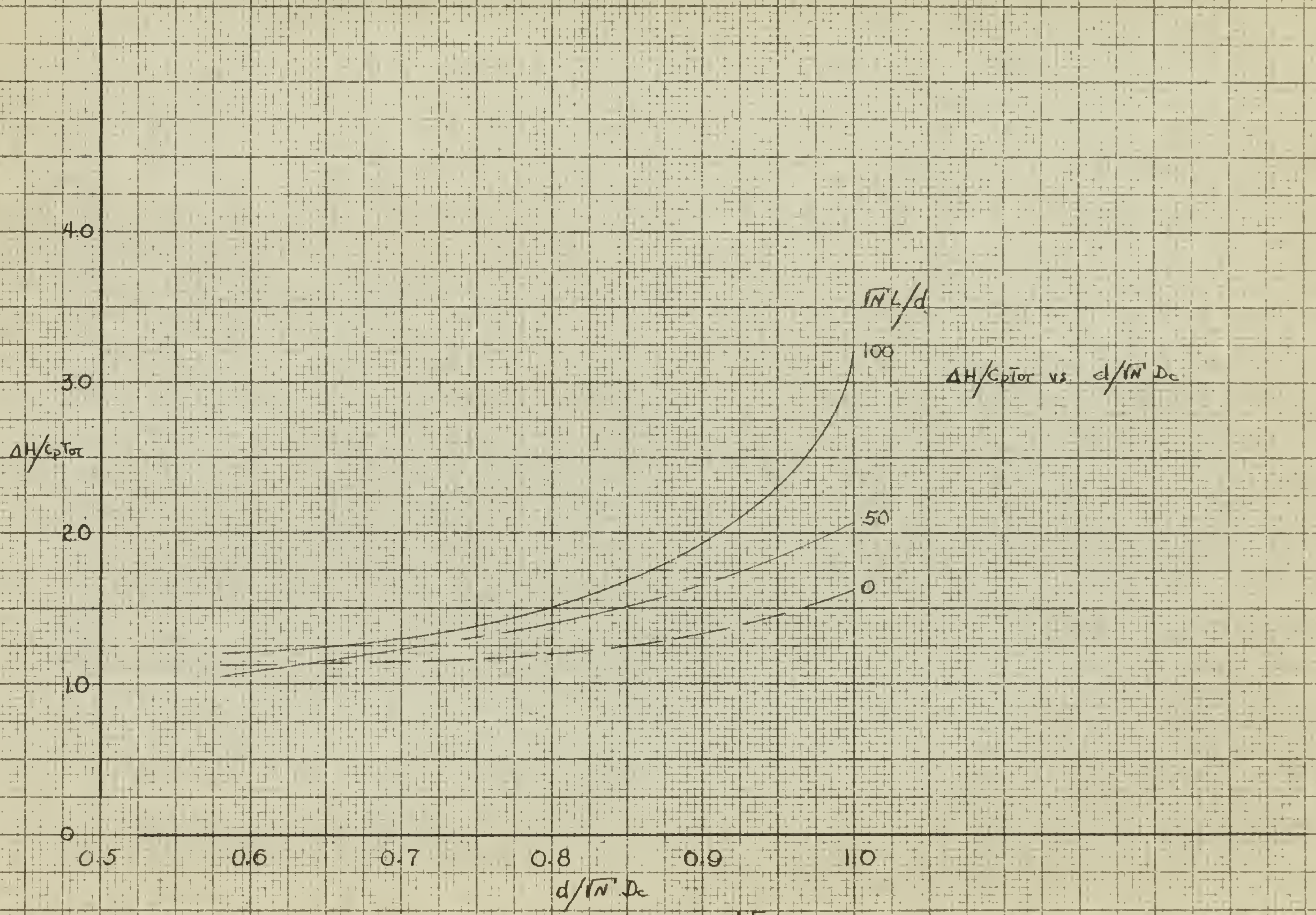
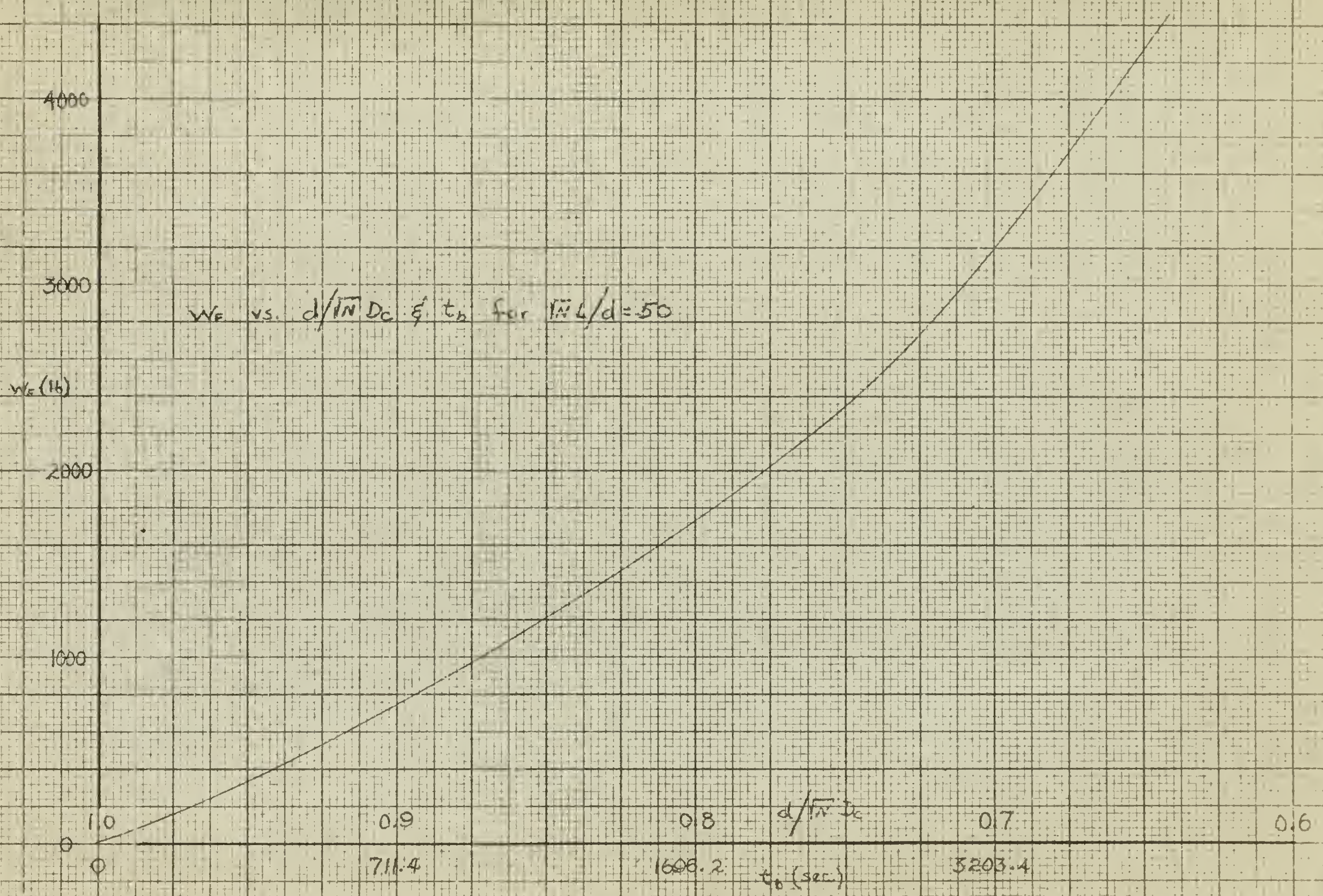

$\Delta H/c_pT_{\sigma}$ vs $d/\sqrt{N}\,D_c$
$\sqrt{N}L/d$
100
50
0
4.0
3.0
2.0
1.0
0
$\Delta H/c_pT_{\sigma}$
0.5
0.6
0.7
0.8
0.9
1.0
$d/\sqrt{N}\,D_c$

FIGURE 15



W_F vs. $d/\sqrt{N}\,D_c$ & t_b for $\sqrt{N}\,L/d = 50$

FIGURE 16

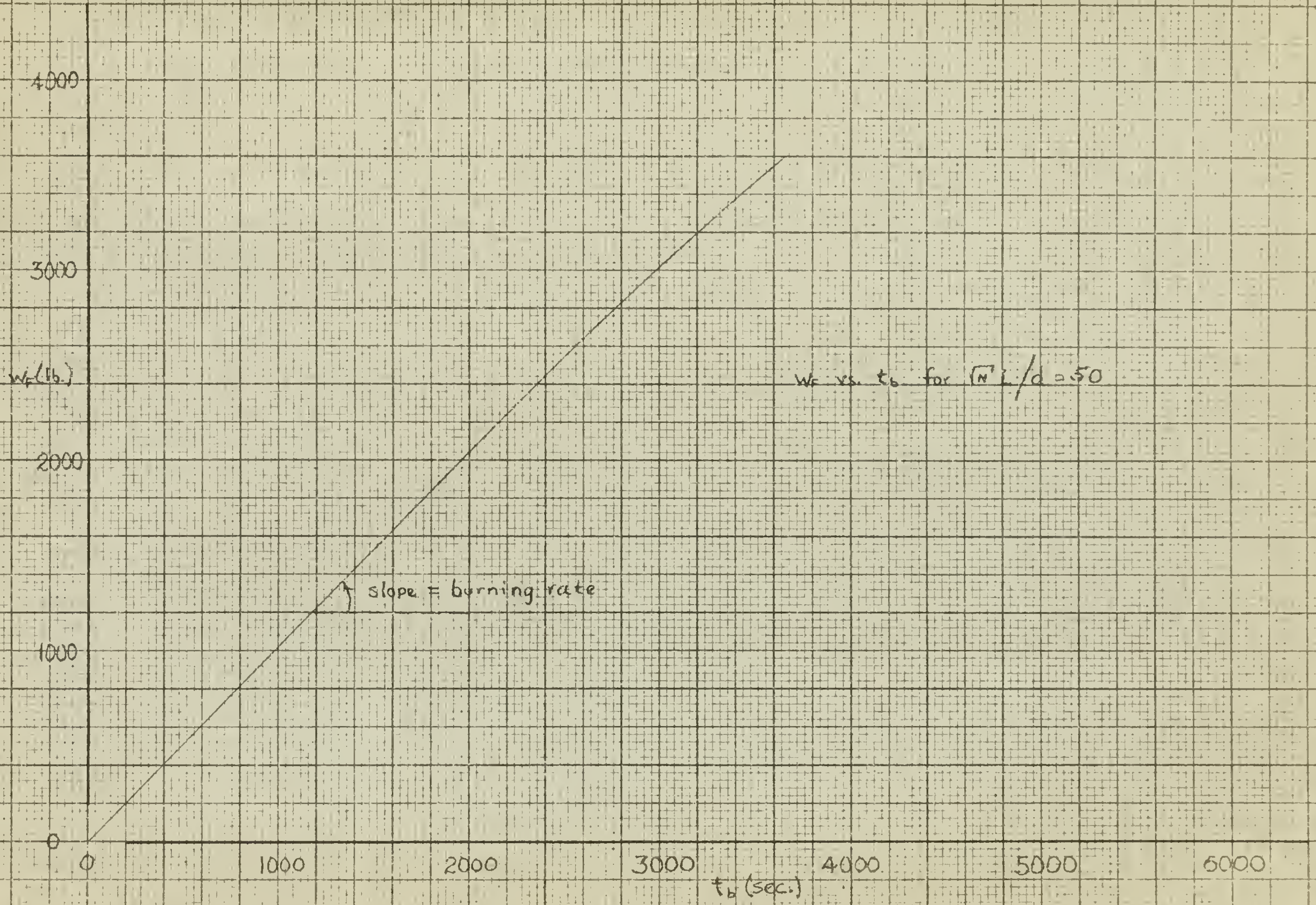
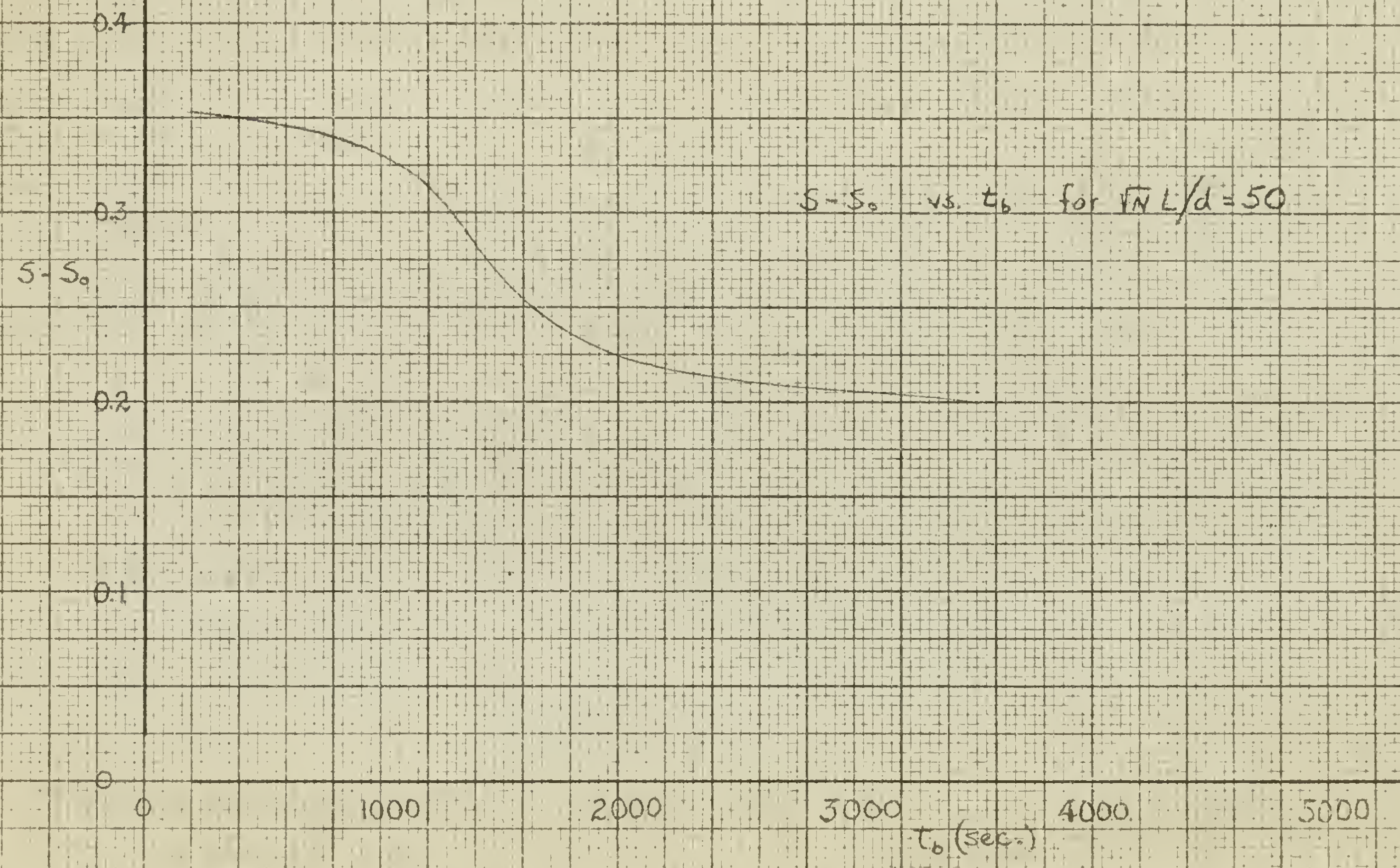

W_f vs. t_b for $\sqrt{N}\,L/d = 50$
slope = burning rate
W_f (lb.)
4000
3000
2000
1000
0
0
1000
2000
3000
4000
5000
6000
t_b (sec.)

FIGURE 17



$S - S_0$ vs. t_b for $\sqrt{N}\,L/d = 50$

FIGURE 18

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